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The stability of unsteady axisymmetric incompressible pipe flow close to a piston. Part 2. Experimental investigation and comparison with computation

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Flow visualization has been used quantitatively to determine the flow relative to a piston and a free surface started from rest. The discharge of water from a cylindrical reservoir was investigated. Flow with a free surface started from rest was found to have a critical Reynolds number (based on tube diameter and surface speed) of about 450 above which a ring vortex was produced just below the surface.

Measurements at Reynolds numbers of 525 and 1200 were compared with computations made by the methods described in Part 1. The computed drift of tracer particles agreed well with observed values. The largest discrepancies occurred in the radial component of the drift in the early stages of the motion and amounted to $2\frac{1}{2}$ % of the tube diameter.

1. Introduction

In this second part of the paper the results of experiments on the flow in front of a piston started from rest are presented and compared with the results computed by the method described in Part 1. The essentially similar problem of the motion near the horizontal free surface of a liquid started from rest in a tube is also investigated.

The only other work on this subject known to the authors is the investigation by Tabaczynski, Hoult & Keck (1970) which was performed at about the same time as the experiments reported here. The creeping flow in the immediate vicinity of the corner is dealt with by Batchelor (1967). The work reported here concerns the Reynolds number range between those of these two treatments. Tabaczynski, Hoult & Keck investigated the transition to turbulence in the ring vortex produced by the starting motion of the piston. Here we consider flows at those Reynolds numbers at which the ring vortex is first formed.

There is a connexion between this work and that of Bellhouse & Talbot (1969) on the aortic valve. In this valve, vortices sited in cavities (sinuses) behind the valve flaps (cusps) play an important part in the action. The present work shows that vortices are formed in the correct sense and position when the flow is started even in the absence of the flaps.

2. Apparatus

Experiments were made with two experimental arrangements. In both, the working section was a Perspex tube of internal diameter 6·26 cm, the tube axis was vertical and water partly filling the tube started its vertical descent from rest. In the first experiments the tube was almost 2 m in length and was supported above a reservoir of the working fluid to which it was connected by copper and

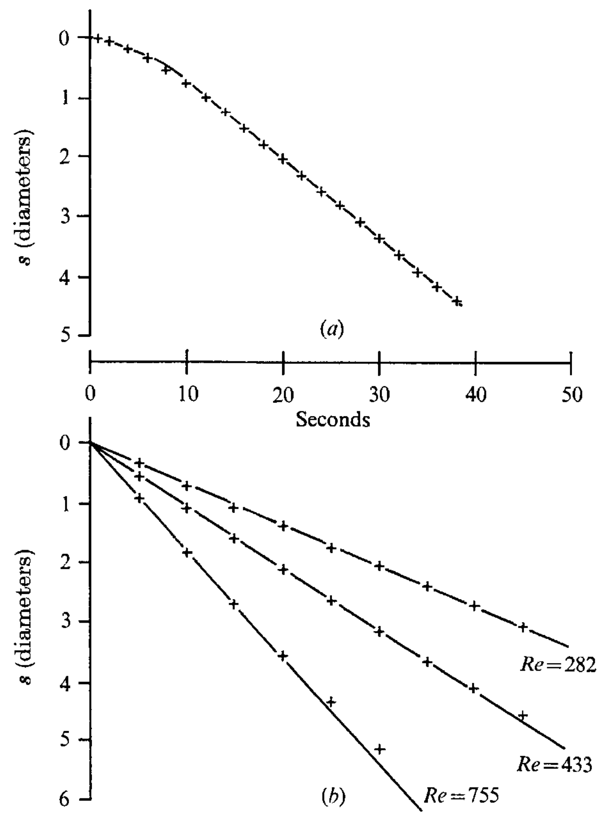


FIGURE 1. Piston and free-surface motions. (a) Solid piston, Re = 525. (b) Free surface.

rubber tubing of 0·63 cm internal diameter. The top of the tube was sealed and connected to a water driven vacuum pump and the working fluid from the reservoir was thus sucked into the working section. A buoyant aluminium piston rested on a stop near the bottom of the tube and was taken up and down with the surface of the liquid. The piston was 6·212 cm diameter and 12·7 cm in length. When it was co-axial with the tube there was thus an annular gap of 0·024 cm. The wires which produced the flow visualization were suspended from the bottom of the piston. The method of measuring velocity is described below.

The working fluid carrying its floating piston was moved down the tube by introducing gas at the top. This method of driving was designed to produce a constant flow independent of time after a starting time which for the present experiments was long. The motion was induced by the introduction of driving gas from high pressure through a sonic throat with the length of outlet tubing chosen to produce a constant flow rate. The details of the driving arrangement are not relevant to the present description.

To facilitate the accurate measurement of the diameter of the working section at the point from which the flow was started the tube was cut at this point. The bottom part, with a wider flexible outlet tube, was then used in another set of experiments. The working fluid was water which descended the tube under the action of gravity when a stopper was rapidly removed from the outlet tube. The speed of fall was varied by constricting the outlet tube with a clamp.

The position of the fluid surface as a function of time is shown in figure 1 for several of the configurations used.

3. Method of velocity measurement

Velocities were determined by measurement of the drift of tracers in the fluid. In the experiments with the floating piston the electrolytic technique pioneered by Baker (1966) was used. The working fluid was an aqueous solution of thymol blue, a pH indicator. The method consists of placing two electrodes in the solution which has been titrated to its end point by the addition of caustic soda solution. When a voltage is applied hydrogen ions migrate to the cathode. There they form molecules which, at the low voltages used, dissolve in the liquid. The excess of hydroxyl ions at the cathode turns the solution there from its acid yellow to dark blue. The electrodes were platinum wires of 25 μm diameter. They were suspended from the piston and kept straight by weights fastened to their free ends. The electrical connexions for the electrodes passed through tubes sealed in the hollow piston and made connexion through air-tight seals in the top of the tube. To prevent the leads from fouling the piston as it moved up the tube in preparation for a run, fine enamelled wires were used. These were kept in tension by a system of pulleys within the tube. The two electrodes were maintained at a potential difference of a few volts for several minutes before the start of a run. In this way coloured fluid without bubbles was produced at the cathode and this had time to diffuse to form a sheath round the wire and drifted with the flow relative to the piston when the motion started. In order to measure more than one component of the motion the cathode was painted with an insulating varnish in bands along its length to produce an interrupted sheath of coloured fluid. The production of a sheath of tracer fluid of very many times the diameter of the wire is significant. If the discoloration is produced after the motion has started it will lie entirely within the wake of the wire and so not follow the motion in the absence of the wire. In the present method of use the wake will produce no effect on the downstream boundary of the sheath, which is the part which is observed.

During the run photographs were taken at accurately determined intervals of about one second. The piston motion and the drift of the discoloured fluid were measured on the negatives in a measuring machine. Corrections were applied for refraction effects by measurements on a millimetre scale inserted in the water in the tube on a separate occasion. Errors were incurred in measuring the position of the corners of the dye bands since the edges of the discoloured fluid became more diffuse as time progressed. The error was estimated by repetition of the readings; the highest departure from the mean of a series of repetitions is indicated on the plotted results in figures 9 and 10.

Qualitative flow visualization studies were made by injecting dilute caustic soda solution into the water through sealed holes in the side of the tube. The results of this and the former method described above can be seen in figure 2 (plate 1).

The arrangement for the second series of experiments was considerably simpler. The tube was open to the atmosphere and before the run the free surface stood within a few centimetres of the top of the tube. Dye was injected from a syringe and photographs taken of its motion as the surface dropped down the tube.

4. Presentation and discussion of results

4.1. *The critical Reynolds number for vortex ring formation*

The initial disparity between the computations of Part 1 and experimentally determined velocities in front of a piston started from rest was found to be due to the appearance of a vortex ring which was not present in the computed velocity field. The occurrence of the vortex was traced, after protracted computational and practical experiments, to an instability of the flow at higher Reynolds numbers. The visualization of the vortex in the flow beneath the solid piston is shown in figure 2. These photographs were taken during an investigation of the cause of the vortex. As the piston changes attitude in the early stages of motion a puff of fluid expelled from the gap could have been responsible for the vortex. Dyed fluid in the gap was found to be drawn into an already existing vortex.

That there is a critical Reynolds number for vortex ring formation is most clearly shown by the simple experiments made on the discharge of water from the cylindrical tube open to the atmosphere at the top. It was seen that vortex rings appeared at high Reynolds number, but not at low Reynolds number. The observations are tabulated below in table 1 and illustrated by figures 3, 4 and 5 (plates 2, 3 and 4). The first photograph of each of the figures was taken before the start of the motion after the dye had become stationary. It appeared that a chaotic initial dye dispersion had no detrimental effect. A screen behind the tube

Surface velocity (cm/s)	Reynolds number	Observations
0·43	282	No vortices
0·66	433	No vortices discernible, but element of doubt
0·80	525	Vortices present
1·01	663	Vortices present
1·15	755	Vortices present
1·83	1200	Vortices present

TABLE 1. Observations of dye near the surface of the water discharged from rest from a vertical tube

had lines marked at 0, 1, 2, etc. diameters below the initial free-surface position. The 'hooked' appearance of the dye patch soon after the start of the motion does not indicate the formation of a vortex ring. The basic program of Part 1 had no closed streamlines but produced this shape, as can be seen from figure 3 of Part 1. Whether a vortex was present or not was decided by observation of the rotation of the dye patches. At a Reynolds number of 633, corresponding to figure 4, there was judged to be a rotation in the vortex of one revolution in one diameter of surface motion after the vortex had formed. At the higher Reynolds number of figure 5, that a vortex has formed is obvious from the still photograph. The rotation of the vortex appeared to decay after a few diameters of motion. At a Reynolds number of 750 the vortex had stopped rotating after four diameters of piston motion and at a Reynolds number of 1200 had almost stopped after the same distance. The instability responsible for the vortex formation may be considered to be the forerunner of turbulence which has its inception at Reynolds numbers of about 2200 in pipe flow. At higher Reynolds numbers more than one vortex was observed: for example, at a Reynolds number of about 2500 two vortex rings were noticed. The one nearer to the piston was in approximately the same position as the single ring discussed above. The lower vortex ring moved downwards and towards the tube axis as the motion progressed.

The instability of the flow relative to the piston, occurring at a turning point of the flow, would seem to be a centrifugal instability like that producing Taylor vortices. The axial velocity profile possesses a maximum which is situated near the wall close to the piston and which is found at smaller radii at larger distances from the piston. The turning flow has a region of decreasing velocity at increasing distance from its centre of curvature and so by Rayleigh's criterion is susceptible to centrifugal instability. It is worth noting that the axial velocity profile has a point of inflexion in the same region.

4.2. *Particle drift in the ring vortex at $Re = 1200$*

Comparison of figure 3 of Part 1 with figure 5 of this part of the paper shows that the basic program will not account for the rolled-up dye streaks clearly visible in the experiments. Calculated particle positions when random disturbances are applied to the computation show a considerably closer agreement, with rolling up of the cloud of particles in a manner closely resembling that observed.

The dye patches at the left-hand side of figure 5, in which the initial dye patch was compact and clearly defined, were chosen for comparison with calculation. The photographs were measured and corrected for refraction by measurement of photographs of a scale inserted in the tube. The outlines of the dye are shown in figures 6, 7 and 8. The initial dye patch is shown in figure 6. The numbered points indicate the initial positions of the particles in a computational study of flow started impulsively to a Reynolds number of 1200. Circles indicate particles in positions where no dye was present. Figures 7 and 8 show the situations after surface movements of 0·8 and 1·75 diameters respectively.

The early stages of the observed motion are characterized by streamers of dye parallel to the wall close to the wall and by what appears to be a jet of dye emanating from close to the corner. The streamers are caused by the rapid motion towards

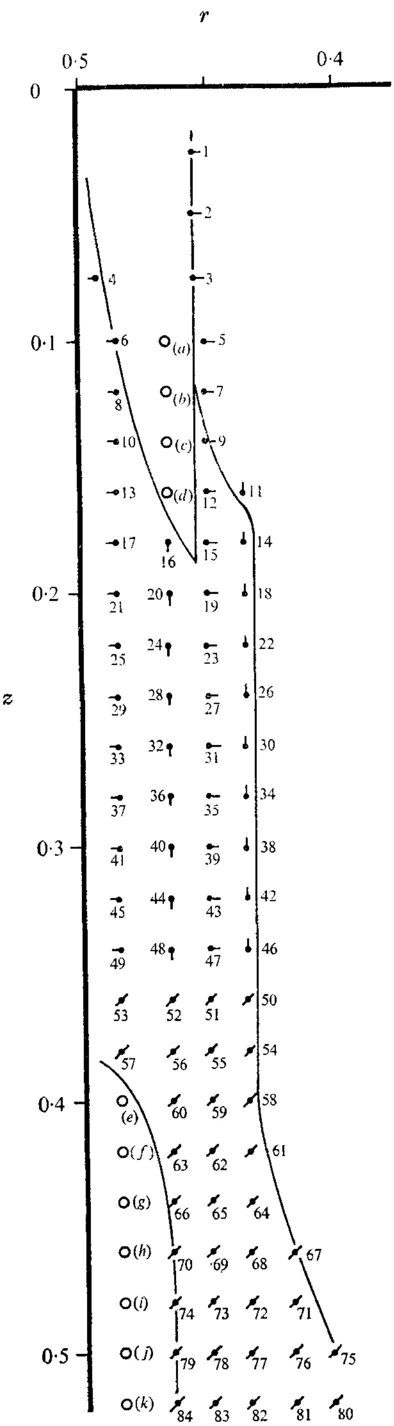


FIGURE 6. Initial dye position from figure 5 and initial particle positions.

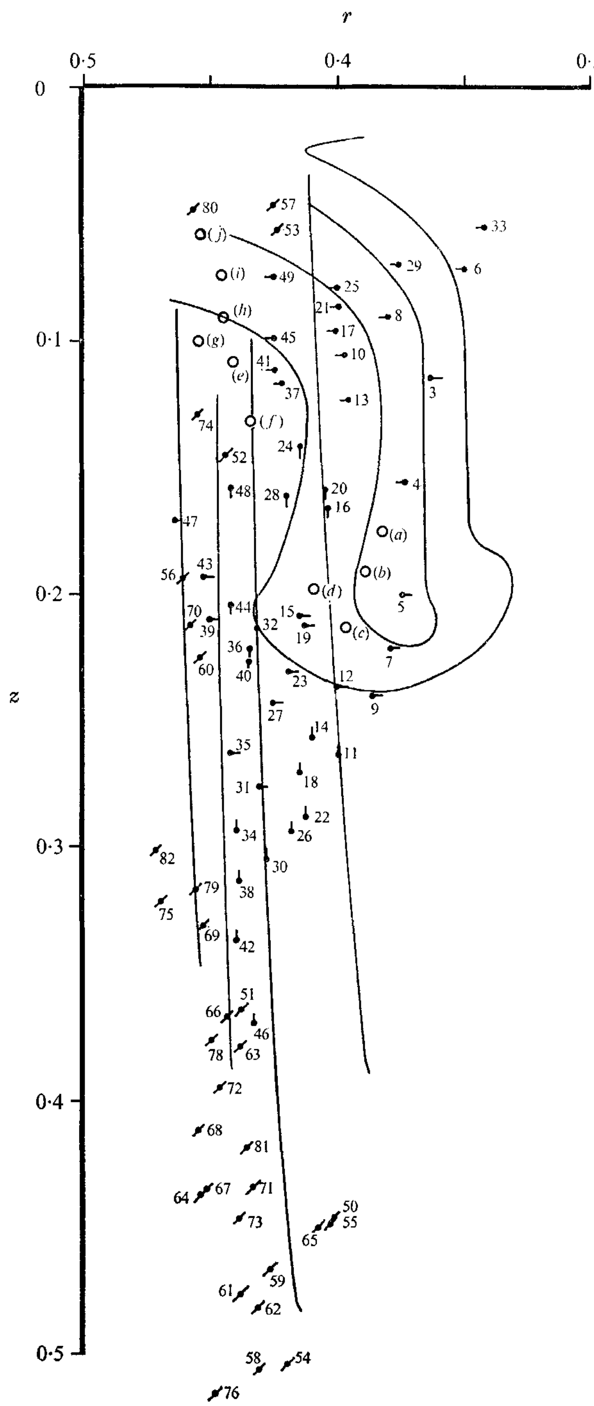


FIGURE 7. After 0·8 diameters of surface motion with an impulsive start to $Re = 1200$. Dye position from figure 5 and computed positions of particles initially as shown in figure 6. Axial mesh = 0·028, radial mesh = 0·020, $\Delta t = 0{\cdot}003$, disturbance amplitude $\psi_0 = 0{\cdot}02$.

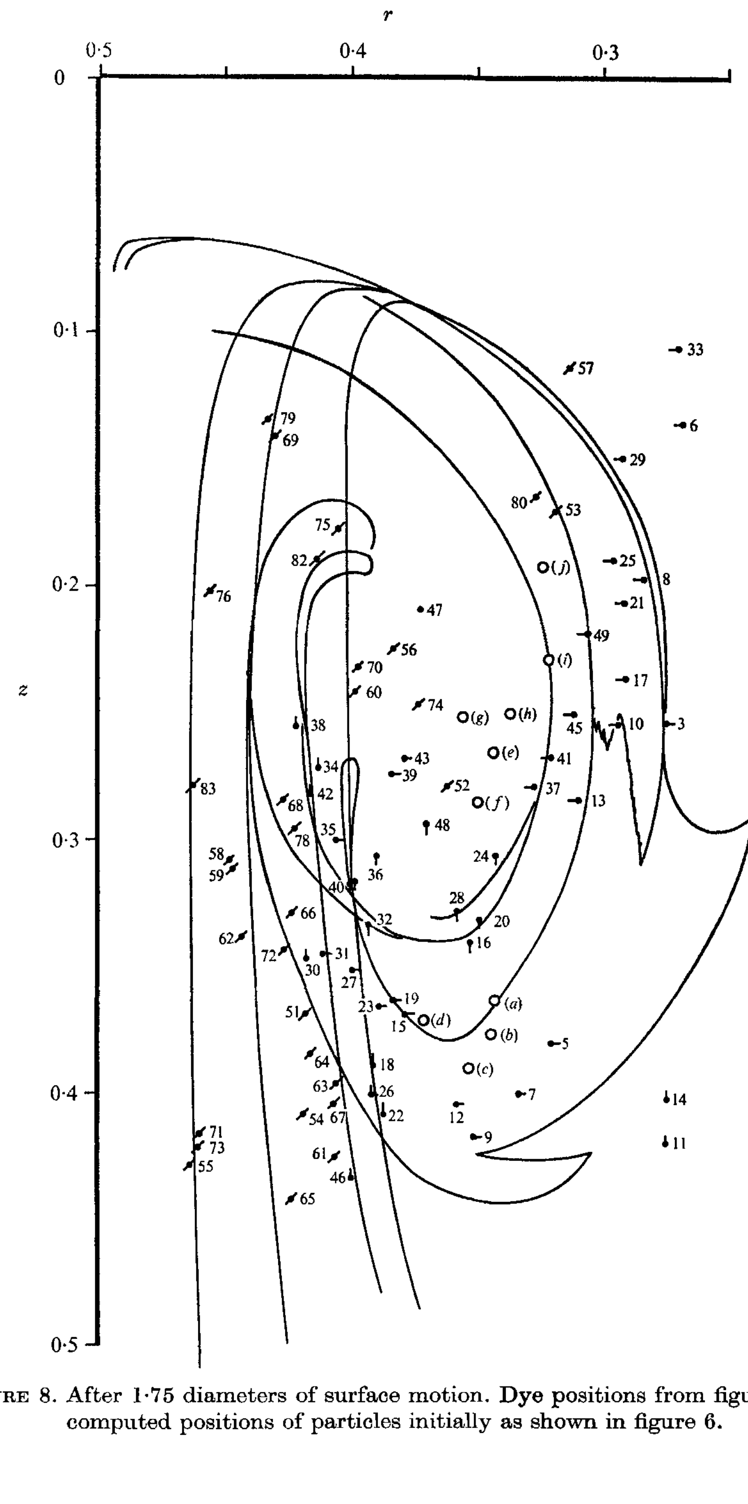


FIGURE 8. After 1·75 diameters of surface motion. Dye positions from figure 5 and computed positions of particles initially as shown in figure 6.

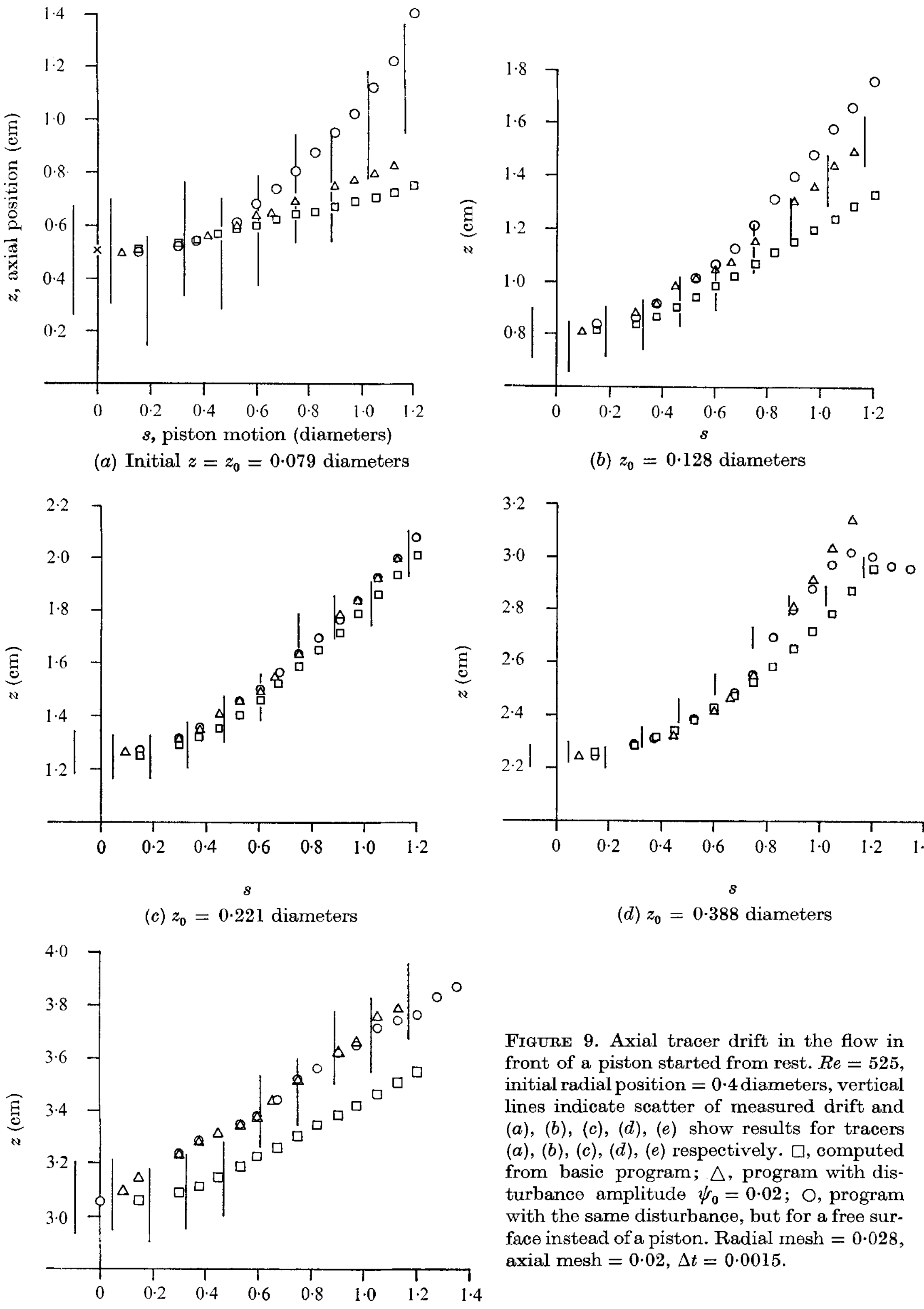
the surface close to the wall and motion away from the surface further out. This is reproduced in the computed motion. The computed particle positions further from the wall have the observed axial values but disagree as far as radial position is concerned, lying a little too close to the wall.

At the later time shown in figure 8 the observed development is much more closely followed by the computed particle positions. Particles are swept up the wall and from there into the interior of the flow following the observed trajectories. The particles away from the wall engage in the observed swirling motion, again on the observed trajectories. The only discrepancy is the lack of particles in the region of the dye patch from which they were absent in the previous figure. The ring vortex appears to be modelled satisfactorily by the randomly disturbed computing program in the later stages of the motion, but in the developing stages of the first diameter of motion the vortex either develops at a different rate or in a slightly different position.

4.3. *Motion of tracer particles at* $Re = 525$

The comparison of computed and experimental results of the last section showed that the ring vortex results from the action of disturbances to the flow. A comparison of calculation and experiment involving more precise measurement of tracer position is presented here for the first 1·2 diameters of piston motion. The tracers followed were the corners of the discoloured fluid produced in bands from the electrode suspended from the piston. The initial radial positions of the tracers were thus approximately the same and equal to four fifths of the radius from the centre of the tube. The initial axial positions of the tracers ranging from 0·079 to 0·534 diameters from the piston surface are indicated on the figures 9 and 10, which show drift distances as a function of time. Measurements were also made at greater distances from the piston. Motion there was almost entirely axial and the results of figures 9(*e*) and 10(*e*) are typical of points further away from the piston. The observed acceleration of the piston was included in the program so that the motion was reproduced in the computing.

Computed points corresponding to the basic program and to the program with random disturbances are shown. For comparison, the results from an identically disturbed program in which a free surface replaces the solid piston are also shown. The calculated axial drift distances agree well with the measured values and disturbances make little difference. The radial drift distances show discrepancies the largest of which is about 2·5 % of the diameter. The introduction of disturbances is seen to produce a significant improvement at some tracer positions. Comparison of the basic program results with observation indicates that the velocity field of a ring vortex in about the observed position would produce agreement. The computations with random disturbance indicate the presence of a ring vortex of about the right strength, but in the wrong position. The results for tracer *c* show the largest difference between computation and experiment. This tracer is most sensitive to vortex position because it lies near the vortex centre. For this tracer the free-surface boundary condition produces agreement with the observed radial drift, presumably because the vortex is in a different position.



(*a*) Initial $z = z_0 = 0{\cdot}079$ diameters

(*b*) $z_0 = 0{\cdot}128$ diameters

(*c*) $z_0 = 0{\cdot}221$ diameters

(*d*) $z_0 = 0{\cdot}388$ diameters

(*e*) $z_0 = 0{\cdot}534$ diameters

FIGURE 9. Axial tracer drift in the flow in front of a piston started from rest. $Re = 525$, initial radial position $= 0{\cdot}4$ diameters, vertical lines indicate scatter of measured drift and (*a*), (*b*), (*c*), (*d*), (*e*) show results for tracers (*a*), (*b*), (*c*), (*d*), (*e*) respectively. □, computed from basic program; △, program with disturbance amplitude $\psi_0 = 0{\cdot}02$; ○, program with the same disturbance, but for a free surface instead of a piston. Radial mesh $= 0{\cdot}028$, axial mesh $= 0{\cdot}02$, $\Delta t = 0{\cdot}0015$.

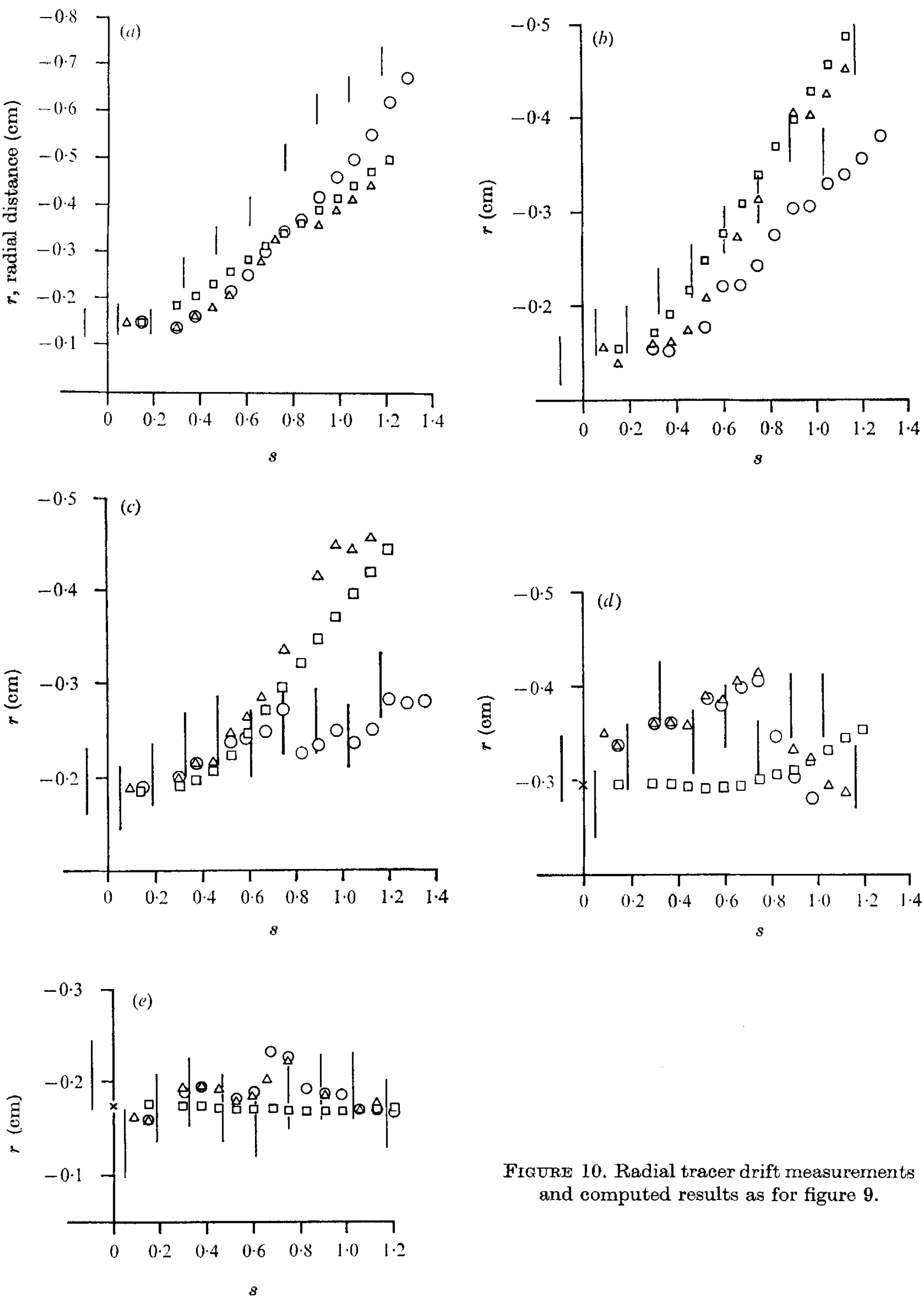


FIGURE 10. Radial tracer drift measurements and computed results as for figure 9.

The conclusions to be drawn from this comparison are the same as those of the last section, namely that the ring vortex is not accurately modelled in its developing phase. There are indications that agreement may improve at later times as was found in the last section.

5. Conclusions

Experiments have been performed on the discharge of water from rest in a vertical cylindrical tube of circular cross-section. In one set of experiments the flow was started almost impulsively and the surface was free and open to the atmosphere. With this arrangement variation of the rate of discharge showed that a critical Reynolds number exists, of value about 450, above which a ring vortex develops during the first diameter of travel. The vortex rotates so that it produces a downward velocity in the centre of the tube. It is positioned close to the walls and at about 0·2 diameters from the surface. At higher Reynolds number (*c*. 2500) more than one vortex was observed.

Experimentally determined drift of tracers in the liquid has been compared with drift in the computed flow field. The instability of the computed flow was initiated by applying random disturbances. Comparison was made with and without these disturbances. The ring vortex resulted only from the disturbed program. In the later stages of the motion when the surface had travelled more than one diameter there was good agreement between observed and computed flows. In the initial stages the position of the vortex differs from that computed. The disturbance applied to the computed flow depends upon the mesh length and time step since random perturbations are applied at each mesh intersection at each time step. In this way the frequencies present in the disturbance are restricted and this may account for the different development of the computed and observed vortices.

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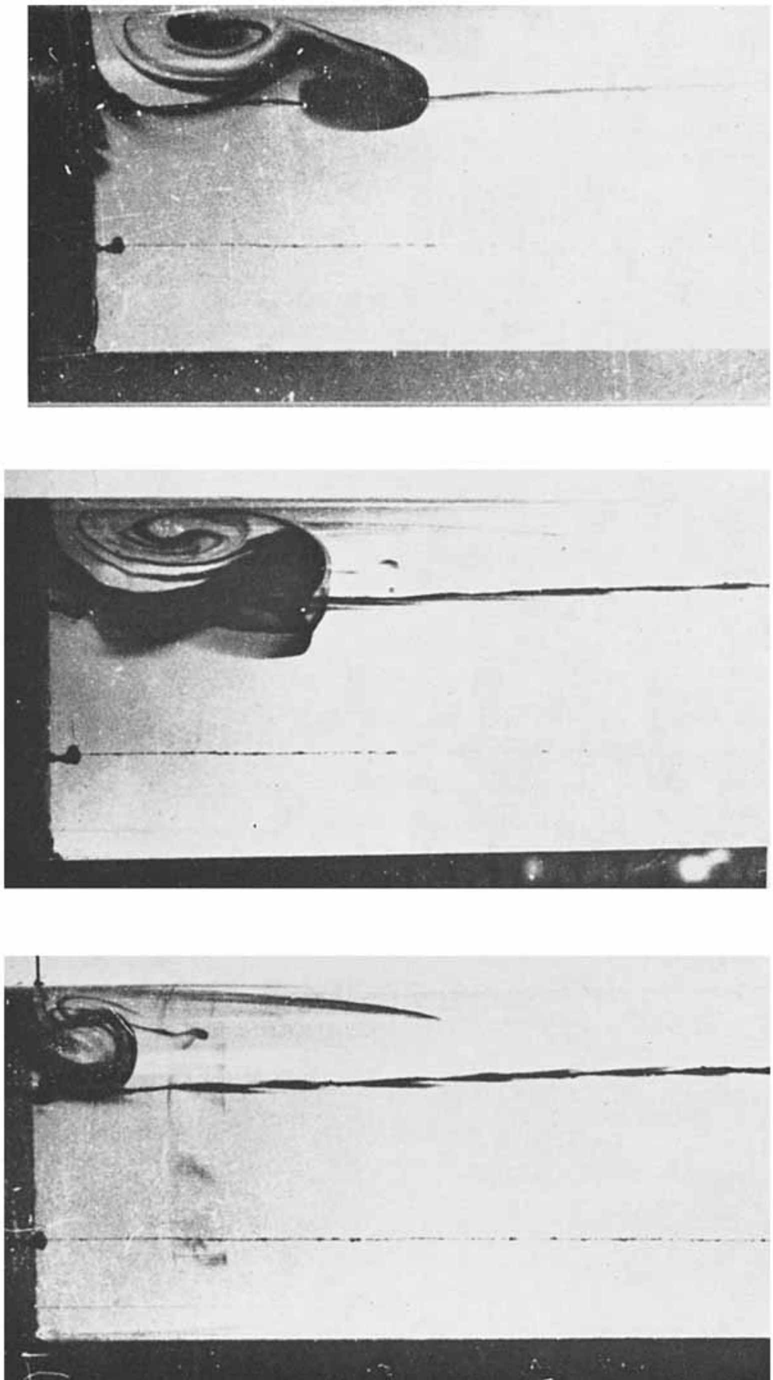
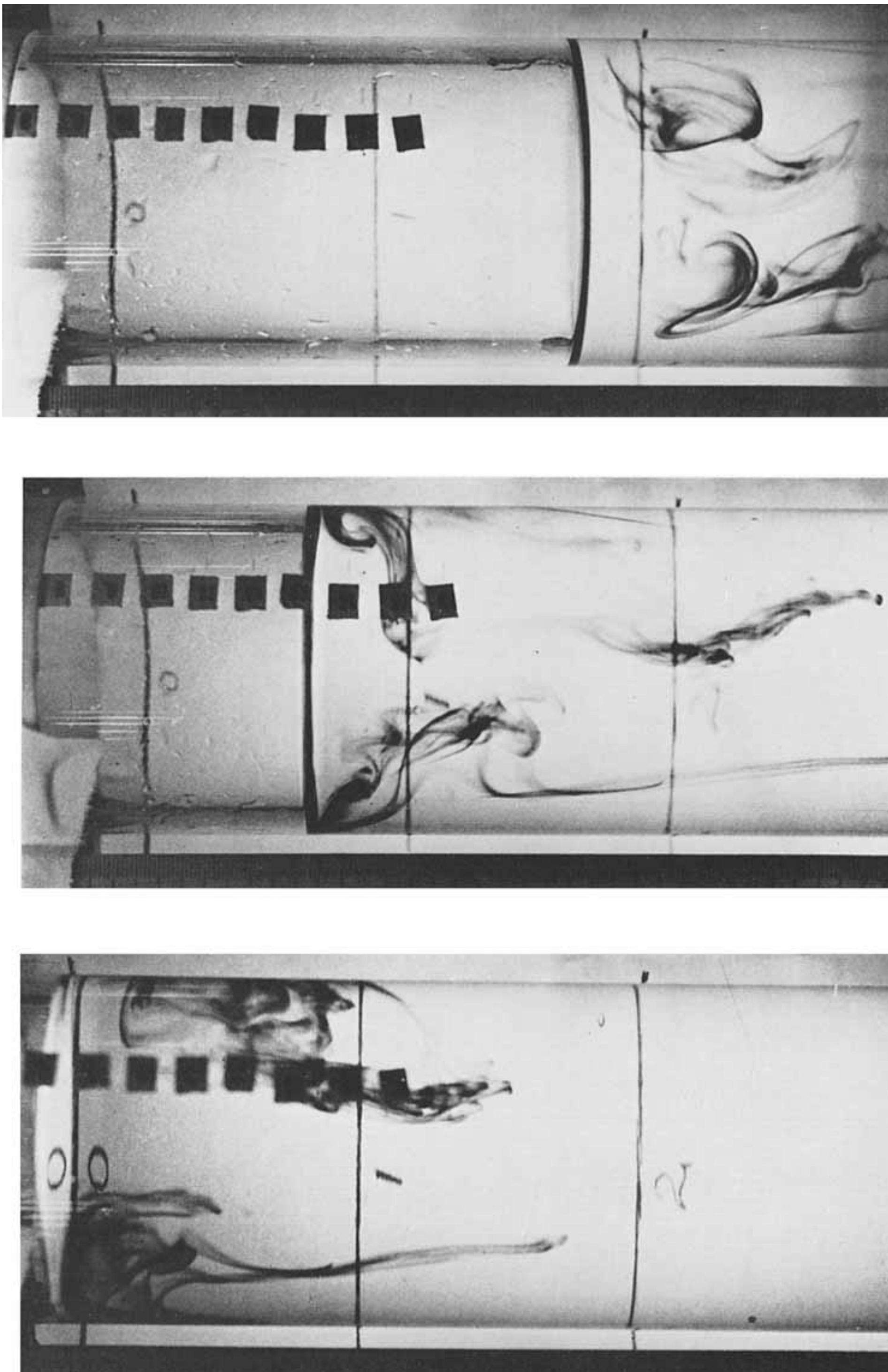


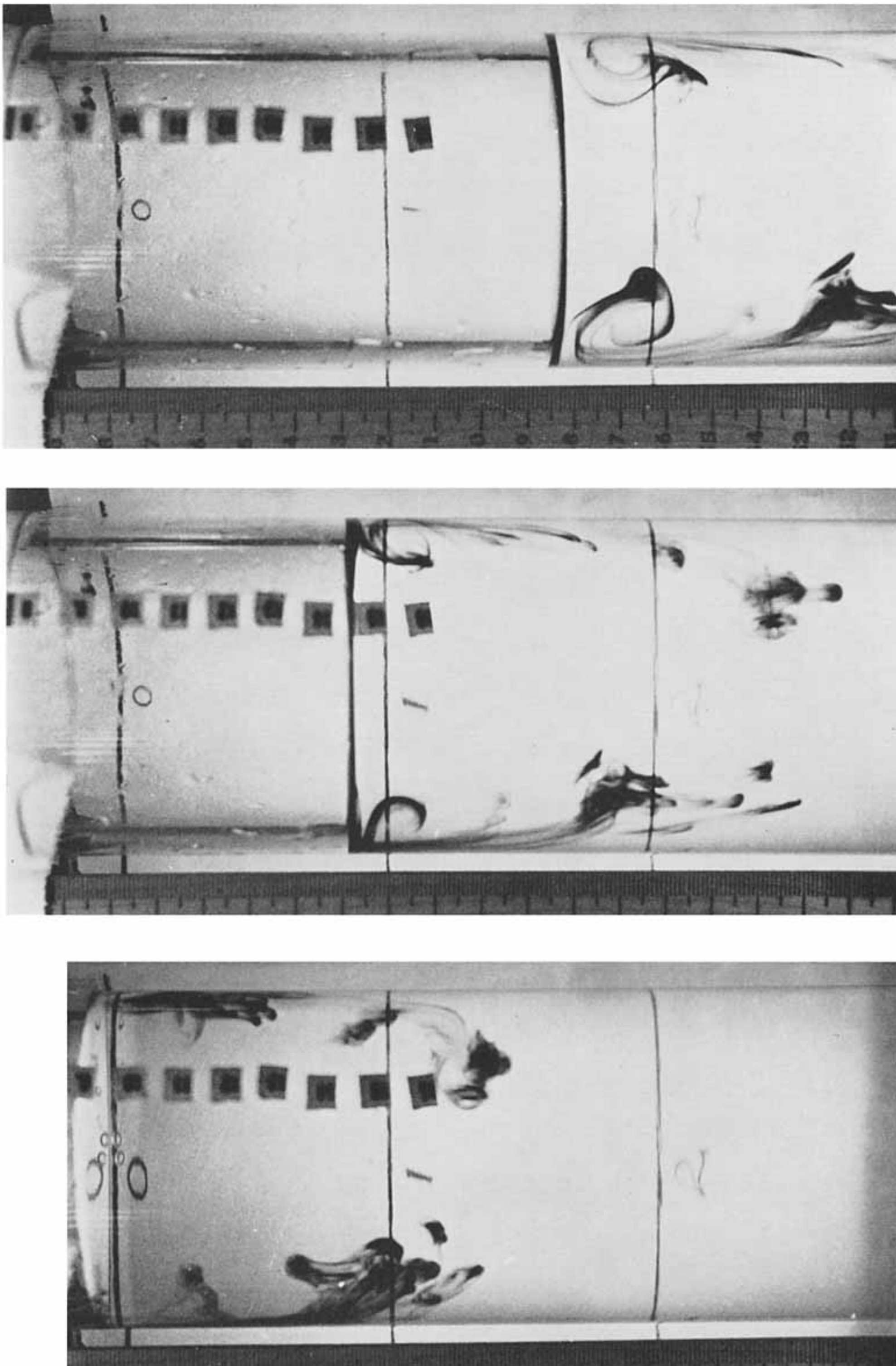
(*a*) (*b*) (*c*)

FIGURE 2. Piston started from rest after 1, 3 and 4 diameters of motion at $Re = 525$. Discoloured fluid produced at the electrode. Dye injected near the wall and in the gap between piston and wall rolls up into the ring vortex.



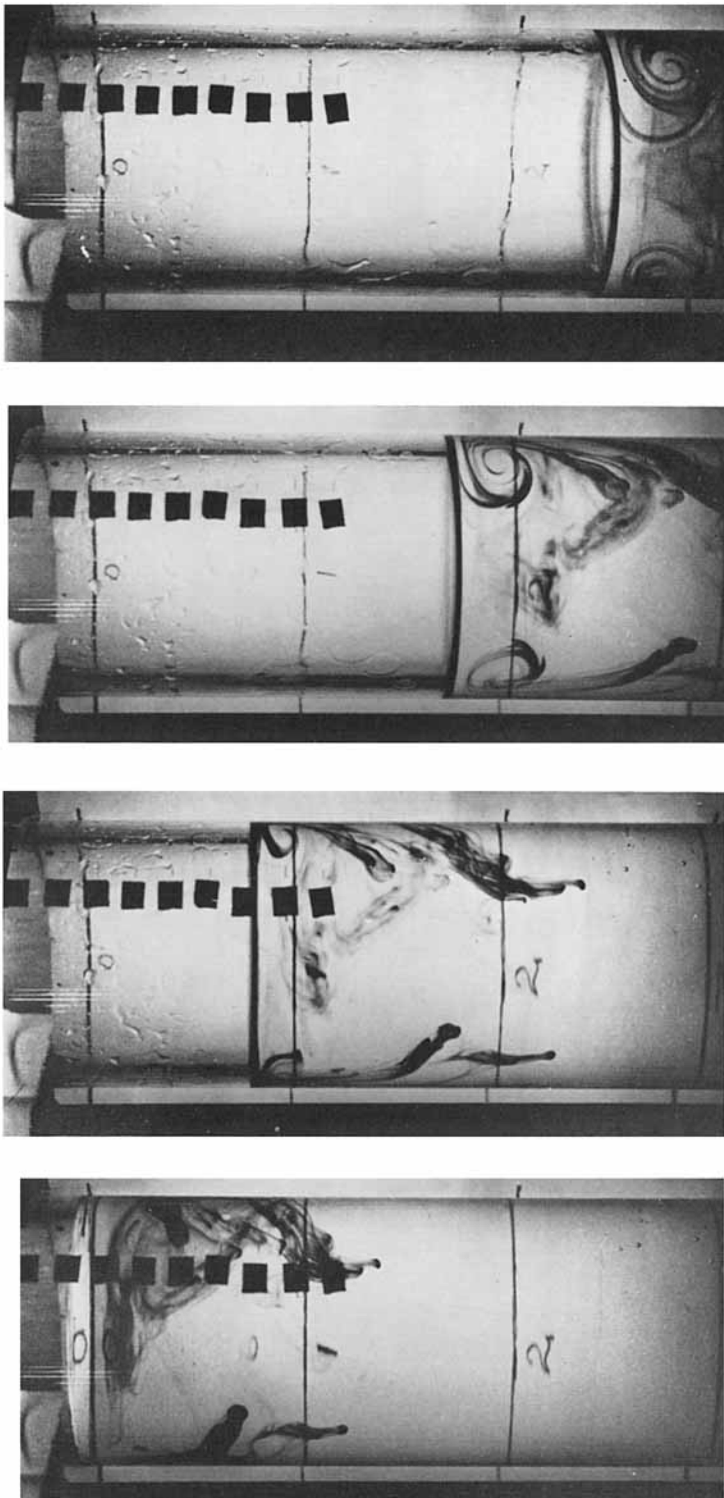
(a) (b) (c)

FIGURE 3. Water flow from rest to a Reynolds number of 433. First photograph before the start of motion.



(*a*) (*b*) (*c*)

FIGURE 4. Water flow from rest to a Reynolds number of 663. First photograph before start of motion.



(*a*) (*b*) (*c*) (*d*)

FIGURE 5. Water flow from rest to a Reynolds number of 1200. First photograph before start of motion.